# Terahertz spectroscopy of semiconductor nanostructures

**Frequency**  $10^5$   $10^6$   $10^7$   $10^8$   $10^9$   $10^{10}$   $10^{11}$   $10^{12}$   $10^{13}$   $10^{14}$   $10^{15}$   $10^{16}$   $10^{17}$   $10^{18}$   $10^{19}$   $10^{20}$ 



Radio and TV Microwaves



viole

ays and

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1 THz  $\leftrightarrow$  1 ps  $\leftrightarrow$  33 cm<sup>-1</sup>  $\leftrightarrow$  0.3 mm  $\leftrightarrow$  48 K  $\leftarrow$ 

# Outline

- Time-domain THz spectroscopy
  - Broadband THz pulses propagating in a free space
- Terahertz response of <u>electrons</u>/holes confined in semiconductor nanostructures ... to the electric field
  - <u>Microscopic</u> conductivity
  - Effective medium approximation (<u>plasmon</u> resonance)
- Experimental results: InP nanowires



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# **Time-domain terahertz spectroscopy**



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#### **Time-domain terahertz spectroscopy**





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# **Time-domain terahertz spectroscopy**



• Retrieval of material parameters

- Time delay =  $2\pi(n-1)d/c \rightarrow$  refractive index *n*
- Attenuation =  $\exp(-2\pi\kappa d/c)$   $\rightarrow$  absorption index  $\kappa$
- Equivalence  $(n+i\kappa)^2 = \hat{\varepsilon} = \frac{i\hat{\sigma}}{\omega\varepsilon_0} \rightarrow \text{complex permittivity, conductivity or carrier mobility can be retrieved <math>too^0 \rightarrow \text{``ultrafast Ohm-meter''}$  [T. Seifert]



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THz beam waist

 $> 300 \ \mu m$ 

Single nanostructure

(< 1 µm)



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#### THz beam waist

 $> 300 \ \mu m$ 



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Ensemble of nanostructures

Benefit of using free-space radiation: we can probe also coupling among nanoparticles

Ensemble of

nanostructures



 $> 300 \ \mu m$ 



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What is the mobility  $\mu$  of charges at a particular frequency f?



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• Frequency *f* versus nanocrystal size *d* 





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# **Calculations of conductivity spectra**



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• Response of free charge carriers – Drude model

Velocity of incident charge carriers

Velocity of scattered charge carriers isotropic scattering

- Relaxation of charge carrier drift velocity

$$\frac{dv}{dt} = -\frac{v}{\tau} + \frac{e_0 E(t)}{m_{\text{eff}}}$$

– Mobility spectrum

$$\mu(f) = e_0 \nu(f) = \frac{e_0}{m_{\text{eff}}} \cdot \frac{\tau}{1 - 2\pi i f \tau}$$

- DC mobility within the Drude model



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- $m_{\rm eff}$  Effective mass of charge carriers
- $e_0$  Elementary charge (1.6×10<sup>-19</sup> C)



• Response of free charge carriers – Drude model



Typical scattering times in semiconductors: GaAs:  $\tau \sim 270$  fs ZnO:  $\tau \sim 30$  fs Fingerprints just in the terahertz spectral region

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• ZnO crystal (bulk material)



- $\rightarrow$  Drude behavior
- $\rightarrow$  Typical free charge carrier response
- $\rightarrow \tau \sim 30 \ fs$



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- $\rightarrow$  Typical free charge carrier response  $\rightarrow \tau \sim 30$  fs

• ZnO nanoparticles





Nanoparticle diameter: ~15 nm

- → Used as electron transporting electrodes in Grätzel solar cells
- $\rightarrow$  There should be free charges
- ★ The response is localized!!!



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- Trouble with fitting THz spectra: too ZnO nanoparticles many models may be employed
  - <del>Drude</del>
  - Drude-Smith
  - Oscillator
  - Hopping
  - Plasmonic model + any response
- List of early works (since 2002): see • references in [J. Photochem. Photobiol. A **215**, 123 (2010)]





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• Frequency *f* versus nanocrystal size *d* 

Illustration from https://www.npr.org/2017/11/11/561217363/terra-incognita-the-planet-factory-and-the-undiscovered-islands?t=1557220165821

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• Lowest quantum transition hf vs.  $E_1 \propto d^{-2}$ 





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• General Kubo formula

$$\sigma = \frac{iNe_0^2}{m\omega} + \frac{1}{\hbar\omega V} \int_0^\infty e^{i\omega t} \operatorname{Tr}(\rho_0[J(t), J(0)]) dt$$



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- Simplifications
  - Single-particle approximation
  - Relaxation time approximation



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Re 
$$\sigma \propto \sum_{k,l} (f_k - f_l) \frac{\left| \langle l | p | k \rangle \right|^2}{(\omega + \omega_k - \omega_l)^2 + \gamma^2}$$



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## **Quantum calculations – relaxation time approximation**

• A charge displaced by the probing electric field relaxes back to its equilibrium position



- Three phases of charge motion
  - Equilibrium

$$\hat{H} = \hat{H}_0 = \frac{\hat{\mathbf{p}}^2}{2m} + V(\hat{\mathbf{r}}) \implies \text{Density matrix } \rho_0 = \sum_k |k\rangle f_k \langle k|$$

- Coherent regime

$$\hat{H} = \hat{H}_0 \underbrace{-e\hat{\mathbf{r}} \cdot \mathbf{E}}_{\Delta \hat{H}} \implies \frac{d\Delta \rho}{dt} = -\frac{i}{\hbar} [\Delta \hat{H}, \rho_0] - \frac{i}{\hbar} [\hat{H}_0, \Delta \rho] - \gamma \Delta \rho$$



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- Three phases of charge motion ullet
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Thermalization regime (diffusion equation with source) —

$$\frac{dn_{\rm th}}{dt} = \gamma \sum_{k,l} \langle \mathbf{r} | k \rangle \langle k | \Delta \rho | l \rangle \langle l | \mathbf{r} \rangle + D \nabla^2 n_{\rm th} + \nabla \cdot \frac{n_{\rm th} \nabla V}{m\gamma}$$
$$-\nabla \cdot \mathbf{j}_{\rm th}$$



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• Total current = coherent + thermalization

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• Example: cube-shaped nanocrystals

$$\sigma \propto \sum_{k,l} \frac{f_k - f_l}{\omega - (\omega_k - \omega_l) + i\gamma} \left[ \sum_{n \text{ odd}} \frac{2D\gamma a S_{kln} x_{kl}}{D\pi^2 n^2 - i\omega a^2} - i(\omega_k - \omega_l) |x_{kl}|^2 \right]$$

- *D* Diffusion coefficient
- *a* Nanocrystal size
- $x_{kl}$  Dipole matrix element  $\langle k | x | l \rangle$
- $f_j$  Occupation of the state j

$$S_{kln} = \delta_{k_y, l_y} \delta_{k_z, l_z} (\delta_{n, k_x - l_x} + \delta_{n, l_x - k_x} - \delta_{n, k_x + l_x})$$



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Large GaAs crystallites  $a = 1 \ \mu m$   $\gamma^{-1} = 270 \ fs$   $D = 180 \ cm^2/s$   $T = 300 \ K$ ....ps://lts.fzu.cz/  $n = 10^{16} \ cm^{-3}$ 





• Lowest quantum transition hf vs.  $E_1 \propto d^{-2}$ 





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• Lowest quantum transition hf vs.  $E_1 \propto d^{-2}$ 



- Lowest quantum transition hf vs.  $E_1 \propto d^{-2}$
- Scattering, thermal broadening

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Ballistic length  $L_{\text{ballistic}} = v_{\text{thermal}}/f = d$ •



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- Models of intrinsic conductivity
  - Starting point: Kubo formula (thermal motion gives the response function; no external field required for the simulations)

$$\mu(\omega) = \frac{e_0}{k_{\rm B}T} \int_0^\infty \langle v(t)v(0) \rangle \exp(i\omega t) \,\mathrm{d}t$$

- Parameters
  - Temperature *T*
  - (Position of the Fermi level)
  - Effective mass, scattering time
- Semi-classical Monte Carlo simulations
  - Evolution of (thermal) velocity v(t) of charge carriers with the statistical distribution f
  - Maxwell-Boltzmann distribution works for high temperatures and low excitation densities
  - Fermi-Dirac distribution needs to be considered otherwise



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• Band-like transport confined by nanoparticle surface



- More sophisticated models including e.g. two types of surfaces
  - Possibility to investigate aggregates of nanoparticles





#### Aggregation of nanoparticles

- Existence of the second length scale appears as the second cut-off frequency



• World of Monte-Carlo semi-classical calculations



#### **Models of electric response: Summary**

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DOI: 10.1002/adom.201900623





# ... 2<sup>nd</sup> part of the story: depolarization fields (effective medium approximation)



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# **Conductivity of complex systems**

- Homogeneous systems
  - local field = applied field
  - no interaction with surface



Charges undergo undisturbed random walk

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- Nanostructured systems
  - local field  $\neq$  applied field



- charges interact with surface
- response to the *local* electric field



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- Effective medium approximation
  - Permittivity profile  $\varepsilon(\mathbf{r})$  is approximated by a single value  $\varepsilon_{eff}$
  - Assumptions: characteristic dimensions << probing wavelength</li>
- Transient permittivity
  - Permittivity  $\varepsilon(\mathbf{r}) + \Delta \varepsilon(\mathbf{r})$  gives a single value  $\varepsilon_{eff} + \Delta \varepsilon_{eff}$
  - Formally:  $\Delta \varepsilon_{\text{eff}} \equiv \Delta \varepsilon_{\text{eff}}[\varepsilon(\mathbf{r}), \Delta \varepsilon(\mathbf{r})]$

## $\Delta \varepsilon_{\text{eff}}[\varepsilon(\mathbf{r}), \Delta \varepsilon(\mathbf{r})]$ is a black-box which is evaluated as follows

- Electric field profile  $E(\mathbf{r})$  is calculated numerically by finite-element method
- Effective permittivity is calculated from energy density:

$$\frac{1}{2}\varepsilon_{\rm eff} \langle \mathbf{E} \rangle^2 = \frac{1}{2V} \int_V \varepsilon(\mathbf{r}) \mathbf{E}^2(\mathbf{r}) dV, \text{ where } \langle \mathbf{E} \rangle = \frac{1}{V} \int_V \mathbf{E}(\mathbf{r}) dV$$



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#### • Implementation

- Steady-state calculations: morphology + permittivities of components = effective permittivity
  - For simple morphologies, special approximations may be applicable (e.g. Maxwell-Garnett for sparse inclusions)
  - Numerical calculations for more complex structures are inevitable



#### Assumptions ullet

- Two-component system; only one component can be photoexcited
- Purely real transient conductivity: Re  $\Delta \sigma \neq 0$ , Im  $\Delta \sigma = 0$  (i.e., purely imaginary transient permittivity)
- Investigated structures •







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• Common properties



• The impact of depolarization fields is encoded in 3 parameters!!!  $(R_{np}, C_{np}, R_p)$ 



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# **Effective medium approximation – summary**

• Effective photoconductivity of inhomogeneous systems



- 3 independent parameters:
  - *V* ... percolation strength
  - *B*, *D* ...non-percolated part
- Describes Maxwell-Garnett
- Conform with Bergman representation
- Pole in the denominator = localized plasmon resonance
- Extensive 2D electrostatic simulations show that this model describes a large number of morphologies including complex percolation pathways
  - Morphology + ground state properties = parameters V, B, and D
- Assumption: components not conducting in the ground state
  - Difference of conductivities needs to be calculated for conducting components. This has been discussed in detail in Joyce et al., Nanotechnol. 24, 214006 (2013).





• Non-percolated photoconductor



Equivalent electrical circuit



(a) Poor conductor (low  $\Delta \sigma_p$ ): Conductive response of particles limits the conductivity

 $\Delta\sigma_{\rm eff} \propto \Delta\sigma_{\rm p}$ 

(b) Good conductor (high  $\Delta \sigma_p$ ): Capacitance limits the conductivity

 $\Delta\sigma_{\rm eff} \propto -i\omega\varepsilon_{\rm 0}\varepsilon_{\rm m}$ 



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3





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## **Conductivity of complex systems**







 $I_{\rm exc} = 300$ 

2

3









# **Effective medium approximation – summary**

• Effective photoconductivity of inhomogeneous systems

$$\Delta \sigma = V \Delta \sigma_{\rm mic} + \frac{B \Delta \sigma_{\rm mic}}{1 + i D \Delta \sigma_{\rm mic} / (\omega \varepsilon_0)}$$

Percolated systemNon-percolated systemLow intensity
$$\Delta \sigma = (V + B)\Delta \sigma_{mic} \approx V\Delta \sigma_{mic}$$
 $\Delta \sigma = B\Delta \sigma_{mic}$ Medium intensity $\Delta \sigma = (V + B)\Delta \sigma_{mic} \approx V\Delta \sigma_{mic}$  $\Delta \sigma = \frac{B\Delta \sigma_{mic}}{1 + iD\Delta \sigma_{mic}/(\omega \epsilon_0)}$ High intensity $\Delta \sigma = (V + B)\Delta \sigma_{mic} \approx V\Delta \sigma_{mic}$  $\Delta \sigma = -i\omega \epsilon_0 \frac{B}{D}$ 

• Assessment of the percolation degree



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# **Experimental results: InP nanowires**



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- InP nanowires well-defined morphology
  - vertically aligned
  - mutually isolated
  - diameter 150 nm, period 500 nm, length 1.3  $\mu$ m
  - *n*-doped by Sn to  $2.5 \times 10^{18}$  cm<sup>-3</sup>



- Charge confinement effects
  - Characteristic parameters
    - Fermi velocity: 10<sup>6</sup> m/s
    - Times: electron scattering (100 fs) probing period (1 ps)
    - $\Rightarrow$  Lengths: 100 nm 1  $\mu$ m  $\gtrsim$  diameter (150 nm)
- Waveguiding of the excitation beam
  - Excitation wavelength  $\approx$  period & diameter  $\Rightarrow$  interferences
  - Analogy: photonic crystal lattice
- Depolarization fields
  - Inhomogeneous system
  - Maxwell-Garnett approximation (low filling fraction)









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Experiment: symbols Theory: lines









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# Summary

Microscopic calculations of THz response of confined charges (semiclassical/quantum) must be combined with effective medium approximation (VBD model applies for a large variety of morphologies)

Info encoded in THz spectra: Nanoparticle size, clustering, bandfilling, presence of further forces, percolation degree

P. Kužel and H. Němec, Terahertz Spectroscopy of Nanomaterials: a Close Look at Charge-Carrier Transport. *Adv. Opt. Mater.* (in press) DOI: 10.1002/adom.201900623







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